Self-Energy and Interaction Effects in Graphene from a Quantum Field Theory Approach

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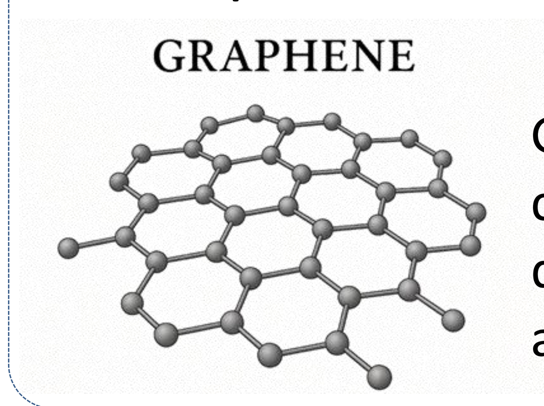
Abstract

- We perform a **Quantum Field theoretical study** of **Dirac fermions in graphene**, focusing on:
  - their electromagnetic interactions
  - emergence of a mass gap via symmetry breaking
- Using an **effective Lagrangian and Dimensional Regularization** (DR), we compute one-loop corrections to the **electron propagator**.
  - A mass term is introduced to model **deviations from graphene's gapless behavior**.
- Our results show momentum-dependent corrections to the **Fermi velocity** and demonstrate conditions under which a **finite energy gap** forms.

Motivation

Our goal is to develop a comprehensive framework that applies ***Quantum Field Theory (QFT) on a spacetime lattice*** *to model, renormalize, simulate, and validate* ***quantum phenomena*** *in* ***graphene-based and other condensed matter systems****.*

- We have employed our software **SUPER_QUANTA** to explore applications in lower-dimensional QFTs, most notably the study of graphene within the framework of **Quantum Electrodynamics (QED) in 2+1 dimensions**. For information on our previous software, **FEDILA**, scan the QR code below:



Graphene is a one-atom-thick sheet of carbon atoms that behaves as a two-dimensional material in many applications.

Theoretical QFT Model

- We extend **Quantum Electrodynamics (QED)** models to explore gap-related phenomena in condensed matter systems (see, e.g., *Phys. Rev. D* **109** (2024) 125014 by M. Bordag et al., *Nucl. Phys. B* **424** (1994) 595 by J. González et al., and references therein). A representative continuum formulation involves the following **Lagrangian density in Euclidean space** (in units $\hbar = c = 1$):

$$\mathcal{L}_{eff} = \bar{\psi}\left(i\tilde{\gamma}_\mu D_\mu - m\, v_F^2\right)\psi\, \delta(z) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu},$$

  - $\psi(x,y,z,t)$: Dirac spinor field, $\tilde{\gamma}_\mu$: modified gamma matrices to incorporate the Fermi velocity v_F, with spatial $\tilde{\gamma}_i = v_F\,\sigma_i$ $(i = 1,2)$ and temporal $\tilde{\gamma}_t = \gamma_t = \sigma_3$, $D_\mu = \partial_\mu - ieA_\mu$: covariant derivative including the interaction with the photon field A_μ, e: electric charge, m: mass term introduced to explore symmetry breaking and potential gap formation, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$: electromagnetic field strength tensor and $\delta(z)$: Dirac delta function, ensuring fermions are confined to the $2D$ graphene sheet at $z = 0$. In the limit $v_F = c$, the momentum integrals appearing in the fermion self-energy, $\Sigma_\psi(q,m)$, are automatically regularized in $D = 3 - 2\varepsilon$ dimensions (ε is the regulator).

Preliminary Calculations ($v_F = c = 1$) and Future Plans

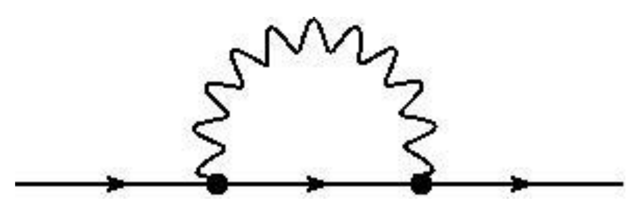
- The **electron self-energy is computed from the Feynman diagram of Fig. 1** in the naïve approximation $v_F = c$ and is given by (ξ is the gauge parameter and $q_\mu = (E, q_x, q_y)$ is the external three-vector energy-momentum in Fig. 1):

$$\Sigma_\psi(q,m) = \frac{e^2}{18\pi q^3}\left[2(\xi+2)\, m\, q^2 \arcsin\left(\frac{q}{\sqrt{m^2+q^2}}\right) + i\,\gamma_\mu\, q_\mu\, \xi\left(m\,q + (q^2-m^2)\arcsin\left(\frac{q}{\sqrt{m^2+q^2}}\right)\right)\right] + O(e^4).$$

- In pristine graphene, the excitations exhibit a linear dispersion relation given by: $E = v_F|q|$, and the quantum corrections to $-i\,\Sigma_\psi(q,0) = \frac{e^2}{16}\xi\begin{pmatrix} E & q_x + iq_y \\ q_x - iq_y & -E\end{pmatrix}\frac{1}{\sqrt{E^2+k_x^2+k_y^2}}$, modify the **Fermi velocity:** $\boldsymbol{v_F \to v_F \pm \frac{e^2}{16}\xi\frac{1}{|q|}}$.
- The **energy gap $\boldsymbol{\Delta E}$** is proportional to the part of the self-energy which is not proportional to $i\,\gamma_\mu\, q_\mu$ at zero momentum:

$$\lim_{q\to 0}\Sigma_\psi(q,m)\Big|_{\gamma_\mu q_\mu = 0} = \frac{e^2}{4\pi}(2+\xi) \Rightarrow \boldsymbol{\Delta E \approx \frac{e^2}{4\pi}(2+\xi)}.$$

- **Future work** regards the computation of quantum corrections in the more realistic case where $v_F \neq 1$, and calculations within lattice regularization to simulate the theory.



One-loop Feynman diagram contributing to the $\Sigma_\psi(q,m)$. A wavy (solid) line represents photons (electrons).

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